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S.I. : ENABLING MATHEMATICAL CULTURES

Enabling mathematical cultures: introduction

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Practice-based philosophy of mathematics. Traditional philosophy of mathematics considers mathematics to be the paradigmatic deductive science; its human practices and cultural variations are seen as mere contingent phenomena that belong into the realm of sociology of mathematics. In this tradition, philosophy deals with a deductive ideal of mathematics that results from abstracting the divergent social phenomena away.

This traditional view has been challenged by philosophers and mathematicians (e.g., Lakatos, 1976; Hersh and Davis, 1981; Asprey and Kitcher, 1988) and the few sociologist that studied mathematics (e.g., MacKenzie, 2004; Heintz, 2000). In the philosophical literature, the philosophers used to be considered as *mavericks* (Mancosu, 2008, § 1).

In the first two decades of the twenty-first century, an increasing number of philosophers of mathematics embraced maverick positions and initiated collaborations with historians, sociologists, scholars of mathematics education, practicing researchers in mathematics, and researchers from other disciplines to study the philosophical significance of social and cultural diversity, human interaction, and variations in mathematical research practices.

Traditional philosophy had deliberately replaced the varied mathematical research practices with an idealised version; sociology had largely avoided mathematics (Heintz, 2000, p. 9); as a consequence, hardly any reliable and representative descrip-

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tion of mathematical research practices based on empirical data existed. This lack of empirical data needed to be remedied, and the community was keen to provide these empirical descriptions as part of international research collaborations such as *Philosophy of Mathematics: Sociological Aspects and Mathematical Practice* (PhiMSAMP; funded by the *Deutsche Forschungsgemeinschaft*; 2006–2010; Löwe and Müller, 2010) and *Mathematical Cultures* (funded by the *Arts and Humanities Research Council*; 2012–2014; Larvor, 2016).

The tools of mathematical research. One particular feature of mathematical research practices that traditional philosophy of mathematics had abstracted away is the multitude of physical, institutional, and computational tools that mathematicians use for their research. These include pen and paper, blackboards and whiteboards, journals and books, lectures and seminar discussions, typewriters and computers, e-mail and preprint servers.[1] The traditional view of philosophy of mathematics is that these tools do not matter for the essence of mathematical research: they are merely part of the context of discovery, not of the context of justification.[2] As recently as 2011, the programme committee of the *Congress on Logic, Methodology and Philosophy of Science* (CLMPS) held in Nancy on the theme of *Logic and Science Facing the New Technologies* was originally reluctant to add a special panel on philosophy of mathematics since some of its members believed that the "new technologies" (i.e., computers) were not relevant for the philosophy of mathematics:

> Mathematicians use their computers every day: they write e-mails, download papers from preprint servers, upload their own research on the same servers, log in to online communities dealing with mathematics to ask questions, they typeset their own papers with the typesetting system LaTeX, etc. But is this use of the computer and the internet relevant for questions of philosophy of mathematics about the nature of mathematics, the relationship between mathematics and the physical world, or the epistemic status of mathematical knowledge? The traditional answer to this question is: Not at all. (Löwe, 2014, p. 399)

In this special issue of the journal *Synthese* entitled *Enabling Mathematical Cultures*, we aim to discuss the rôle of these tools enabling mathematical research by studying the social and cultural features of mathematical research practices and their philosophical ramifications.

The 2017 Oxford workshop & the project *Social Machines of Mathematics*. Our special issue grew out of a workshop held in Oxford from 5 to 7 December 2017 that celebrated the successful completion of the project *Social Machines of Mathematics* led by Ursula Martin and funded by the *Engineering and Physical Sciences Research Council* (EPSRC). In stark contrast to the traditional views reflected in the mentioned deliberations of the CLMPS programme committee, this project was rooted in the

[1] The use of these tools has been studied, e.g., by Greiffenhagen (2014) and Barany and MacKenzie (2016) in the case of blackboards; Heintz (2000), Gerovitch (2016), and Lane (2017) in the case of research seminars; Mihaljević et al. (2016) and Mihaljević and Roy (2019) in the case of publication patterns and keynote lectures; or Merz (1998) and Merz and Knorr-Cetina (1997) in the case of e-mail collaborations.

[2] For a discussion of various problems with Reichenbach's context distinction in philosophy of science, cf., e.g., Hoyningen-Huene (1987), Schickore and Steinle (2006), and Ammon (2011).

observation that mathematical research practice is currently at a remarkable inflexion point, with new technologies, e.g., in the form of crowdsourcing, symbolic computation, and proof verification, radically extending the power and limits of individual mathematicians (with a particular emphasis on the Polymath projects, MathOverflow and the relevance of the arXiv). The concept of *social machines*, a new paradigm considering a combination of people and computers as a single problem-solving entity identified by Berners-Lee and Fischetti (2000, p. 172), gave the project its name.[3] The 2017 workshop widened the scope from computational tools and *social machines* in the sense of Berners-Lee and Fischetti to include all research tools traditionally excluded by philosophers of mathematics, including physical tools (e.g., paper folding), institutional tools (e.g., published papers and mathematical abstract services), and social tools (co-authorship and the rôle of the audience).

The special issue consists of seven papers of which five were presented at the Oxford workshop in December 2017 (the schedule of the workshop is printed at the end of this introduction). The workshop was organised by Joe Corneli, Lorenzo Lane, Ursula Martin, and Fenner Tanswell with crucial administrative support by Sarah Baldwin from the Department of Computer Science at the University of Oxford. The guest editors wish to express their sincere thanks to Baldwin, Corneli, Lane, and Tanswell for their contribution to the success of the workshop.

Is it philosophy or not? Due to the lack of empirical data on mathematical research practices, a large part of the work in the field of practice-based philosophy of mathematics is devoted to the collection of empirical evidence of practices (in the form of case studies, data collection from archives and internet resources, interviews, or even cognitive experiments). With such an emphasis on descriptive empirical work, the question has been raised whether practice-based philosophy of mathematics might not rather be a form of empirical social science of mathematics than philosophy of mathematics proper.[4] The complicated relationship between empirical work and philosophy of mathematics and its repercussions for the thorny topic of what the research area should be called[5] was discussed at length by Löwe (2016); it was argued that the link to philosophy of mathematics stems from the fact that many of the researchers in the field are "motivated by philosophical questions" (Löwe, 2016, p. 40) and that these links provide an embedding in the wider philosophical discourse. Several papers in this special issue provide these important links to important philosophical debates (in particular, in epistemology).

The papers of the special issue. In the following, we shall give a brief overview of the content of the seven papers that form part of this special issue and how they

[3] Cf. Martin and Pease (2013), Martin (2015, 2016), Corneli et al. (2017a, b), and Pease and Martin (2018) for more details on and results of the project.

[4] E.g., Jullien and Soler (2014, p. 228; emphasis in the original): "[T]he [...] approaches are not, strictly speaking, approaches 'in the philosophy of mathematical practice'. They are, rather, [...] *non*-philosophical perspectives on mathematical practice that are *used* by philosophers of mathematical practice or, more prudently, on which *some* philosophers of mathematical practice *can find relevant* to rely."

[5] "Other terms than 'philosophy of mathematical practice' have been used [...], among them 'empirical philosophy of mathematics', 'practice-based philosophy of mathematics', '(socio-) empirically informed philosophy of mathematics', or 'philosophy of real mathematics'. (Löwe, 2016, p. 31, fn. 6)"

relate to the overarching theme. The papers fall into three groups: the papers that discuss mathematical writing and publishing (the papers by Andersen, Johannsen, & Sørensen, Ashton, and Barany), the papers that discuss various particular practices (Johnson and Rittberg & Friedman), and the papers that link issues of research practices to epistemological questions (Inglis & Mejia-Ramos and Weber).

The paper *Mathematicians Writing for Mathematicians* by Andersen, Johannsen and Sørensen discusses the process of revising a research paper in order to take the intended audience into account. The discussion is based on an empirical interview study highlighting one particular case, the revision of a paper by a talented doctoral student co-authored with his experienced supervisor.

Ashton's paper *The Role of Audience in Mathematical Proof Development* continues the theme of writing with an audience in mind from the first paper. Starting from a traditional position that claims that the deductive nature of mathematics makes mathematical writing independent of audiences, Ashton argues that this is not a correct description of the mathematical writing process. She provides another case study (from knot theory) as empirical evidence.

Barany's programmatic paper *Abstract Relations: Bibliography and the Infrastructures of Modern Mathematics* goes beyond the research practice of mathematical writing and considers the importance of scientific abstracting in mathematics. Barany argues that systematic scientific abstracting played a crucial rôle for the nature of mathematicians' research and theories, placing the mathematical abstract collections (such as *Zentralblatt für Mathematik* and *Mathematical Reviews*) in their historical context within the first half of the twentieth century and then examining their consequences and legacies for the second half of the twentieth century and beyond.

In the paper *Some Examples of the Role of Finance in Enabling Mathematics*, Johnson discusses the interplay between extra-mathematical practices (in this case, financial practices) and the development of research mathematics. The paper's main contribution is the description of the significance of financial practice in validating the *Dutch Book Argument*, the most popular justification for subjective probabilities.

Rittberg and Friedman consider the practice of paper folding in their paper *The Material Reasoning of Folding Paper* and consider the question whether it constitutes a mathematical practice. This question and their findings are particularly interesting from the point of view of challenging traditional views in epistemology: mathematical paper folding constitutes a material reasoning practice generating knowledge that is largely non-propositional.

In their paper *Functional Explanation in Mathematics*, Inglis and Mejia-Ramos approach a very traditional question in the philosophy of mathematics: the relationship between proof and explanation. They discuss Wilkenfeld's concept of *functional explanation* and argue that typical philosophical accounts of mathematical explanation are derivable from Wilkenfeld's proposal.

Finally, Weber's *The role of syntactic representations in set theory* discusses the relevance of the syntactic form of mathematical statements for the insights that mathematicians gain from them. He links his findings to the discussions of arguments against Azzouni's *Derivation Indicator View*.

Schedule of the workshop *Enabling Mathematical Cultures* held in Oxford, December 2017.

Tuesday 5 December 2017.

12:30–14:00 Registration.
14:00–15:00 Ursula Martin: *The social machine of mathematics*.
15:05–15:35 András Máté: *Hungarian mathematical culture: different interests, common features*.
15:35–16:10 Tea and Coffee.
16:10–16:55 Michael Barany: *Abstract Relations: Media, Social Structure, and Sociable Structuralism in Modern Distributed Mathematics*.
17:00–17:45 Lorenzo Lane: *Socialising Mathematical Social Machines: Exploring the Transformative Role of Web Technologies*.
19:00–21:00 Dinner at Somerville College.

Wednesday 6 December 2017.

09:30–10:00 Marcos Cramer: *Modelling Arguments about Foundations of Mathematics in Structured Argumentation Theory*.
10:05–10:35 Henrik Kragh Sørensen, Line Edslev Andersen, & Mikkel Willum Johansen: *The practice of framing mathematical papers: Training to write to convince*.
10:40–11:10 Keith Weber: *Mathematical discourse among set theorists on the relationship between derivations and proofs*.
11:10–11:25 Tea Break.
11:25–11:55 Colin Rittberg & Michael Friedman: *Paper Folding as a Mathematical Culture*.
12:00–12:40 Fenner Tanswell: *Proof, Rigour and Mathematical Virtues*.
12:40–14:00 Lunch.
14:00–14:40 Dave Murray-Rust: *Towards an argumentative understanding of mathematical discourse*.
14:45–15:30 Matthew Inglis: *A Cognitive Account of Mathematical Explanation*.
15:35–16:20 Andrew Aberdein: *Redefining Mathematical Revolutions*.
16:20–17:00 Tea Break.
17:00–18:00 Alan Bundy: *Automated Reasoning in the Age of the Internet*.
19:00–21:00 Informal Dinner at St Anne's College.

Thursday 7 December 2017.

09:30–10:00 Slava Gerovitch, Julia Braverman, & Anna Mirny: *CrowdMath: Massive Research Collaboration among High School and College Students*.

10:05–10:35 Nick de Hoog: *Linked structures of collaboration*.
10:40–11:10 Gila Hanna: *Connecting two different views of mathematical explanation*.
11:10–11:40 Tea Break.
11:40–12:25 Joe Corneli: *Intelligent machinery via social machines: a proposal*.
12:30–13:15 Alison Pease: *Empirical Studies of Online Mathematics*.
13:15–14:45 Lunch.
after 14:45 Informal discussions.

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